1. Introduction

In this Memoriam paper, we review the impact of the scientific contributions of Erol Başar, focusing our attention on the paradigm shift in the study of brain dynamics: from linear systems to random non-linear systems analysis and the new vision of the brain as a dynamical system. We will argue that this is an example of a “scientific revolution” as defined by T. Kuhn in The Structure of Scientific Revolutions, (Kuhn, 1970). Kuhn proposed that science regularly transits through different stages (Fig. 1). “Normal science” usually adheres to what Kuhn called scientific revolutions that are characterized by a change in the paradigm of the scientific community. These revolutions are marked by a shift in the fundamental assumptions and methods of research, leading to a new understanding of the phenomena under study.
3. Model Crisis. Demonstration of severely nonlinear phenomena in EEG

4. Model Revolution. Recognition of severely nonlinear phenomena in the EEG

1. Normal Science. Description of Brain processes using only linear models

2. Model Drift. Demonstration of interaction between background and evoked activity

5. Paradigm Change. Description of Brain Dynamics as nonlinear

Fig. 1. Yearly Number of Erol Başar’s peer-reviewed scientific publications on data of WoS ISI Web of Knowledge, n.d.). These are restricted to the field of neurosciences (period 1972–2018) with an indication of the institutions where he worked.

“paradigm”: a set of generally accepted theories providing standard concepts and tools that focus and constrain research efforts. At a certain point, the paradigm increasingly fails to solve particularly worrisome puzzles called “anomalies”—“model drift”. This produces a “model crisis” in the specific discipline, in which confidence in the status quo is progressively lost, leading to the emergence of a “model revolution” in which a new paradigm is established.

Erol Başar was one of the pioneers of the “Nonlinear Brain Dynamics” scientific revolution. This conclusion is based upon a scientometric analysis of his scientific production, evaluating its impact on a wide network of collaborators and research groups. We will discuss the impact of his results in the introduction of random non-linear analysis methods in neuroscience. As we shall see, this impact was a consequence of his more general belief and practice of applying the methods of innovative physics and mathematics to explain complex processes in the brain. He was deeply influenced by the work of N. Wiener, for which he was primed by his education by the pioneers of modern physics.

This paper is a selective review organized as follow: In Section 2, we present the methodology used for the scientometric study and the main obtained results, in Section 3 we include an analysis about the most important contributions of Erol Başar to neurophysiology, a summary of some of the most influential papers of this author and a discussion about his legacy and transcendence in neurophysiology.

2. Scientometric analysis of Erol Başar’s scientific production

Scientometrics is a discipline that performs quantitative studies of science and technology using mathematical, statistical, and data analysis methods and techniques (Hirsch, 2005). Here we apply these techniques to the study of the scientific contributions of Erol Başar.

Our data was downloaded from the Web of Science (WoS) (“ISI Web of Knowledge,” n.d.). We used two search strategies: #1: AU = (Başar, E NOT (Başar-Eroglu, Canan OR Başar, Ertugrul OR Başar-Eroglu, C OR Başar-Eroglu, C OR Başar-Eroglu, Canan)) AND OG = (ISTANBUL KULTUR UNIVERSITY OR DOKUZ EYLUL UNIVERSITY OR UNIVERSITY OF LUBECK OR HACETTEPE UNIVERSITY OR UNIVERSITY HANNOVER), #2: AU = ((Başar, Erol OR Başar, E) AND (Başar-Eroglu, Canan OR Başar-Eroglu, C OR Başar-Eroglu, Canan OR Başar-Eroglu, C OR Başar-Eroglu, Canan)) AND OG = (ISTANBUL KULTUR UNIVERSITY OR DOKUZ EYLUL UNIVERSITY OR UNIVERSITY OF LUBECK OR HACETTEPE UNIVERSITY OR UNIVERSITY HANNOVER). The results of both search strategies were combined in a single set. The data was downloaded from the WoS on April 5th, 2019.

The active life of Erol Başar’s publishing in neurosciences covered a period of 46 years (1972–2018); he died in 2017 but some papers were published in 2018, in which he worked in four universities (Hacettepe University, Turkey (1970–1979), University of Lubeck, Germany (1980–1999), Dokuz Eylul, Turkey (2000–2006) and Istanbul Kultur University, Turkey (2007–2017). We have found three articles published in the period 1967–1969. (Başar and Weiss, 1967; Başar et al., 1968; Başar and Weiss, 1968) but these were focused on the analysis of the frequency response of pressure-induced changes in the flow resistance (in coronary vasculature of heart and kidneys) in rats. Two of these publications were on Pflügers Archiv (today Pflügers Archiv: European Journal of Physiology), which was the first physiological journal of the world. We have excluded them from our analysis for this paper considering that these three publications were not related directly to the neurophysiology field. However, they already showed his interest in physics and signal processing applied to biology.

The number of papers per year and institution is reported in Fig. 2. The total production of peer review papers was 278. This gives an average of 6.04 papers per year. Even though the scientific production in the first 25 years was half of this average per year (2.68), in this period he produced many of the most insightful and influential contributions to neurosciences (Başar, 1972a; Başar, 1983; Başar-Eroglu et al., 1992; Başar et al., 1997; Başar et al., 1987). His WoS h-index was 49. Also, he published an important number of monographs (7) and edited 12 books related to the main fields of his research and other hot topics such as the meaning of the mind concept and natural philosophy. We just highlight some of these monographs: EEG-Brain Dynamics: EEG-brain dynamics: Relation between EEG and Brain evoked potentials (1980), (Başar, 1980) Brain Dynamics: Progress and Perspectives, (Başar and Bullock, 1989) Brain Function and Oscillations: Brain Oscillations: Principles and Approaches (1998; Başar, 1998) and Brain-body-mind in the Nebulous Cartesian System: A Holistic Approach by Oscillations (2011) (Başar, 2010a). Finally, he was a prominent organizer of scientific conferences around the world. He chaired 11 international conferences and congresses, including the First International Conference on Machinery of the Mind (1989) (John, 1989a, 1989b) held in Havana and Istanbul 2006 the world congress of International Organization of Psychophysiology (IOP).

The total number of citations with and without self-citations was 7500 and 3650, respectively (Fig. 3 a) and b)). Superimposed is the cumulative number of citations per paper per year. The main peaks of the citations (Fig. 3 b) are between 10 and 17 years after the fundamental contributions on overcoming the artificial distinction between evoked potentials and spontaneous activity. The elapsed time is understandable due to the usual inertia in understanding and applying new concepts. The next peak in the citations graph was at the end of his career, reflecting an increasing recognition by the scientific community of Başar’s contributions. Interestingly, the peaks in the number of citations coincide with the periods of greater scientific production. As can be seen by the cumulative curves of the number of citations per publication by year, his influence increased exponentially.

The widespread worldwide distribution of citations to Başar’s articles is reported in Fig. 4, with 3428 citations distributed in 73 countries. The US, Turkey, and Germany are nodes in this map. Note that these scientists formed a tight network, see figure (Fig. 5) for the network of citation links between countries.

He was not only highly cited; he was also a node for collaborations. Researchers of 12 countries were co-authors of his articles (Fig. 6). We highlight the collaborations with authors from Turkey and Germany, countries where he developed the main part of his scientific career. The 10 most cited collaborators of Başar are also reported in Table 1, including the number of publications and h-index. In Table 2 we present the 10 most cited scientific papers of Erol Başar (citations included in the WoS database). Of these most cited papers, 90% were published in the period 1992–2001. All of them are related to the main keywords of his contribution in the neurophysiology, they are (brain, neural) oscillation...
or oscillatory. The list does not include very important papers with a more specific or technical scope, particularly those where he critically discussed the role of the physics theory and mathematics in neurophysiology (and neurosciences, in general) and other contributions included in monographs. Finally, for Table 2, it is interesting to discuss briefly some correlations between coauthors included in Table 1 here. Among the top 10 most cited scientific articles of Erol Başar, all of them have as coauthors his most important collaborators. Başar-Eroğlu, C., Schürmann, M., Karakas, S., Kolev, V.

From all the above, we conclude that Erol Başar was not only a prolific scientist but also a very influential one.

Based on the scientometric study of Başar’s publications we conclude that:

- The topics of most cited publications (see Table 2) are related to the Başar’s main field of research: Brain Dynamics. This is evident if we consider keyword used in the titles (e.g. Brain oscillations, oscillatory responses, Gamma, alpha, delta, and theta oscillations)
- The widespread worldwide distribution of coauthoring and citations of Başar’s shows the significance and depth influence of his research in neurophysiology. In that context, the collaboration with world-leading groups shows the reciprocal impact of these researchers in the development of brain dynamics.
- The scientific production of Başar’s career covers 47 years (1972–2018). It is a critical period in the development of brain dynamics and it is approximately the same time interval in that developed their academic work, significant researches in the brain dynamics field (e.g. W. J. Freeman (1927–2016), E. Roy John (1924–2009), Fernando Lopes da Silva (1935–2019))

### 3. Brain dynamics and Erol Başar’s key contributions

#### 3.1. The roots of Erol Başar’ scientific views

At the beginning of the twenties, new theories in Physics modified classical concepts such as time and space (Special Relativity) and certainly about the state of a particle (Quantum Mechanics). These theories led to a new paradigm in Physics. Here we use the term “paradigm” as developed by Kuhn (1970). In this context, it was logical that the interpretation of the phenomena and processes in the brain requires the application of different concepts and theories inspired both by classical and new (relativistic and quantum) physics.

Probably due to the “inertia” to these paradigm changes, the impact of these novel physical theories to study brain functioning was a challenge for neuroscientists in the past century. Many defended that only statistical thermodynamics together with nonlinear mechanics were the correct underpinning for brain theories. This led to these authors ignored the potential applications of other physical theories (Walter and Yates, 1981; Yates, 1980).

Fortunately, novel ideas flourished in applied fields of Mathematics (Nonlinear Systems Theory, including chaos) triggered novel approaches to modeling in Physics, Biology, and Engineering. In this context, the pioneering works of Wiener (Wiener, 1958; Wiener, 1948) were decisive in the study of physical systems. He was the first scientist to describe the deep relationship between technical processes and living organisms. Wiener was the founder of a school with was continued by Ilya Prigogine (the father of the “dissipative structures”) (Nicolis and Prigogine, 1977), Haken (Synergetic) (Haken, 1977; Haken, 1983) and Thom (Catastrophe Theory) (Thom, 1973).

It against this background that the professional career of Erol Başar (1938–2017) elapsed: the change of paradigm in Physics, the birth of new fields in Applied Mathematics, and therefore innovative approaches to study the brain. He was particularly attuned to this atmosphere since some of the most important physicists of the past century contributed to his basic formation in theoretical Physics.

Regarding brain science, a profound influence was his close collaboration and friendship with T. H. Bullock, whose work in comparative neuroscience. This was an example of interdisciplinary research with a profound integration of knowledge of the central nervous system for varied species, developmental stages, and states (Bullock, 1984b) and (Bullock, 1984a). Since 1982, for 25 years, they established a deep relationship as reflected in s books they edited jointly. (Başar-Eroğlu et al., 1992; Başar and Bullock, 1989). This collaboration delved into nonlinear EEG dynamics in different animal species. In the 80s and 90s, Bullock’s research team in California and Başar’s group in Lübeck conducted a series of comparative research on invertebrates and lower vertebrates focusing their attention on the effect of acetylcholine, dopamine, and noradrenaline (Schütt, Başar, & Bullock, 1992).

Last, but not least, Başar was very influenced by philosophers such as Bergson (Bergson, 1907) All these influences converged in his open and
Fig. 3. Geographical distribution of the 3428 citations to Erol Başar’s articles in 73 counties. The US, Turkey, and Germany are nodes in this map.
creative mind.

In the delightful monography “Brain-body-Mind in the Nebulous Cartesian System: A Holistic Approach by Oscillations” (Başar, 2010a), a book published in 2011, 6 yes before his decease, Başar explained the aspects that influenced him to write this book (It is in a certain sense a resume of his scientific career). We just mention the first two of them:

- **Brain oscillations:** This topic became a critical branch of neurosciences, mainly since the 1970s. It is based on the study of the oscillatory patterns of neural activity.
- **Quantum mechanics and Fundamentals of Natural Philosophy:** According to Başar, Quantum Mechanics introduced an unavoidable
element of randomness into his theoretical interpretations of brain research.

Concerning the most influential scientist in his career, he mentioned W. Heisenberg. This influence was in two ways, scientifically and personally. Heisenberg is the father of the very famous uncertainty principle (Heisenberg, 1927) in quantum mechanics. It is one of the cornerstones of the new physics and modified forever the deterministic landscape of the Newtonian physics contained in the Principia (Newton, 1687). Heisenberg was a mentor of the young Erol in his first studies in Germany. Another critical influence in his early career was Professor Carl F. Weizsäcker, an eminent physicist and philosopher, a pupil of

Table 1
Top 10 collaborators of Erol Başar using as a reference to the number of published articles.

<table>
<thead>
<tr>
<th>Author</th>
<th>Articles</th>
<th>h-index</th>
<th>Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Güntekin, Bahar</td>
<td>102</td>
<td>20</td>
<td>Istanbul Medipol Univ</td>
</tr>
<tr>
<td>Başar-Eroğlu, Canan</td>
<td>59</td>
<td>36</td>
<td>Univ Bremen, Inst Psychol &amp; Cognit Research</td>
</tr>
<tr>
<td>Schürmann, Martin</td>
<td>49</td>
<td>29</td>
<td>Dokuz Eylul Univ</td>
</tr>
<tr>
<td>Yener, Gorsev G.</td>
<td>30</td>
<td>13</td>
<td>Dokuz Eylul Univ</td>
</tr>
<tr>
<td>Ozerdem, Aysegul</td>
<td>25</td>
<td>17</td>
<td>Dokuz Eylul Univ</td>
</tr>
<tr>
<td>Karakaş, Sirel</td>
<td>24</td>
<td>21</td>
<td>Hacettepe Univ</td>
</tr>
<tr>
<td>Tulay, Elif</td>
<td>21</td>
<td>8</td>
<td>Istanbul Kultur Uni</td>
</tr>
<tr>
<td>Emek-Savaş, Derya</td>
<td>19</td>
<td>9</td>
<td>Dokuz Eylul Univ</td>
</tr>
<tr>
<td>Demiralp, Tamer</td>
<td>16</td>
<td>29</td>
<td>Istanbul Univ</td>
</tr>
<tr>
<td>Kolev, Vasil</td>
<td>12</td>
<td>36</td>
<td>Bulgarian Acad.Sci</td>
</tr>
</tbody>
</table>

Table 2
Top ten most cited scientific articles of Erol Başar.

<table>
<thead>
<tr>
<th>Title</th>
<th>Year of publication</th>
<th>Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma, alpha, delta, and theta oscillations govern cognitive processes (Basar et al., 2000)</td>
<td>2000</td>
<td>597</td>
</tr>
<tr>
<td>Wavelet entropy: a new tool for analysis of short duration brain electrical signals (Hosono et al., 2001)</td>
<td>2001</td>
<td>469</td>
</tr>
<tr>
<td>P300-response: possible psychophysiological correlates in delta and theta frequency channels. A review (Basar-Eroglu et al., 1992)</td>
<td>1992</td>
<td>338</td>
</tr>
<tr>
<td>Brain oscillations in perception and memory (Basar et al., 2000)</td>
<td>2000</td>
<td>255</td>
</tr>
<tr>
<td>Alpha oscillations in brain functioning: An integrative theory (Basar et al., 1997)</td>
<td>1997</td>
<td>226</td>
</tr>
<tr>
<td>Are cognitive processes manifested in event-related gamma, alpha, theta, and delta oscillations in the EEG? (Basar et al., 1999)</td>
<td>1999</td>
<td>218</td>
</tr>
<tr>
<td>The Associations Between 40 Hz-EEG and the Middle Latency Response of the Auditory Evoked Potential (Basar et al., 1987)</td>
<td>1987</td>
<td>134</td>
</tr>
<tr>
<td>A new strategy involving multiple cognitive paradigms demonstrates that ERP components are determined by the superposition of oscillatory responses (Karakaş et al., 2000a, 2000b)</td>
<td>2000</td>
<td>130</td>
</tr>
<tr>
<td>The genesis of human event-related responses explained through the theory of oscillatory neural assemblies (Karakaş et al., 2000a, 2000b)</td>
<td>2000</td>
<td>127</td>
</tr>
</tbody>
</table>

Before the ‘70s the dominant view in electrophysiology was that of absolute separation between “evoked” and Background activity—instrumentation blinded us.

a) Dawson cathode-ray photographic superposition technique to identify a Visual Evoked Potential. EEG recordings stimuli aligned to its presentation. From (Ciganek, 1964).

b) The Computer of Average Transients (CAT 400C) used by one of the authors (PAVS) in the ‘70s to extract the supposedly constant Evoked response following linear theory.

Fig. 6. The instrumental basis for the separation of Evoked and Background EEG activity.
Heisenberg. He stimulated Erol to focus his research in physiology. No less important was the influence of Wiener whose *Cybernetics* (*Wiener, 1948*), published in 1948 was an inspiration for the birth of brain dynamics, and whose work on random nonlinear dynamics was so prescient (*Wiener, 1958*). Finally, the holistic research of Başar comprised many influences from Albert Einstein through three important concepts: synchrony of clocks, the interpretation of Brownian motion, and unconscious problem solving (*Başar, 2010a*). Başar also acknowledged the imprint of classical thinkers such as Newton, Leibniz, Darwin, Galileo, Descartes, Pascal, Hume, Locke, and others (*Başar, 2010a*).

His early studies in Germany gave him a solid preparation in mathematics and theoretical physics. This formation led him to explore the application of physical theories to explain the physiology of the brain (*“Toward a physical approach to integrative physiology I. Brain dynamics and physical causality”*, and references therein; *Başar, 1983*). He focused the attention particularly on the problem of causality in physics and brain theories. Several physical effects were analyzed by Başar following the question addressed by Walter and Yate: (*Walter and Yates, 1981; Yates, 1980*) *“Why should neuroscience look the physics for its theories?”*. In this sense several effects were considered by Başar:

- The determinism of Newtonian Mechanics
- Statistical Mechanics
- Electromagnetism
- Quantum Mechanics

One of the most novel proposals of Başar was that the responses of the brain are not predictable in the Newtonian sense: they obey a

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**Fig. 7. Interaction of Event-Related and ongoing EEG activity.**

A simple example of this would be squaring a sine wave input to produce an output of twice the frequency according to David et al. (2005).
probabilistic density function and are statistically and partially predictable. As a first step, he proposed a model of coupled nonlinear oscillators to fit roughly the relationship between evoked responses and spontaneous activity of the brain. Additionally, Başar highlighted the brain responses depict indeterministic phase-jumping (probabilistic oscillator) and frequency stabilization mechanisms. Other physical analogies were considered among them: laser theories, magnetic response susceptibility, and dissipative structures.

3.2. Linear and nonlinear brain dynamics

To understand the nonlinear brain dynamics “scientific revolution”, we must first characterize the signal analysis paradigm in human electrophysiology predominant until the first half of the 20th century. The main subjects studied were two types of brain activity. One was the spontaneous, quasi-periodic activity discovered by Berger known as the 3.2. Linear and nonlinear brain dynamics

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response susceptibility, and dissipative structures.

3.2. Linear and nonlinear brain dynamics

To understand the nonlinear brain dynamics “scientific revolution”, we must first characterize the signal analysis paradigm in human electrophysiology predominant until the first half of the 20th century. The main subjects studied were two types of brain activity. One was the spontaneous, quasi-periodic activity discovered by Berger known as the Electroencephalogram (EEG) (Berger, 1929). On the other hand, the electrical responses of the brain known as “evoked potentials”—though well known in animal or human anesthetized experiments—were obscured by the ongoing EEG which, considered many neuroscientists as “noise”. With the advent of, first the photographic superposition technique of Dawson (1954), and then the special purpose computer of Clynes (1962) the idea of the Evoked potential as a deterministic response to an external transient, independent and superimposed on random background oscillations, became firmly entrenched as a scientific paradigm (Fig. 7).

This dichotomy of Evoked response and background EEG dovetailed very well with the prevalent theories and methods for signal analysis of linear systems. Considering the brain as a dynamical linear system brings to the forefront analysis of the EEG and ERP in the frequency domain. Based on the Fourier Transform which had been elegantly developed by Wiener in the 1930s (Paley et al., 1934).

As shown in Fig. 7a the first estimates of this response were historically obtained by superposition, grounding the idea of the evoked response as a fixed deterministic signal $\pi(t)$ summed to a random background activity $\beta(t)$ with $0 \leq t \leq T$ the time elapsed after the stimulus at $t=0$. A single EEG trial response to a stimulus was therefore thought to be:

$$v_i(t) = \pi(t) + \beta_i(t) \tag{1.1}$$

The computer of average transients is based upon precisely upon this assumption (Fig. 7b) to calculate the usual estimator of $\pi(t)$ as the average of the $v_i(t)$ trials. This made a lot of sense if the brain were a linear system. EEG responses to a stimulus would thus be the result of the sum of the inputs to such a linear system that is of a pulse-like transient $\delta(t)$ at the time of each stimulus and independent background noise $\beta_i(t)$. The output would be the result of passing the input

$$y(\omega) = h(\omega) \cdot \delta(\omega) + \beta(\omega)$$

through a linear transfer function $h(t)$:

$$v_i(t) = h^*(t) \cdot \delta(t) + \beta_i(t) \tag{1.2}$$

Now let us look at the Fourier transform of the system output. This is known as the Output Frequency Response (OFR). Note that we denote the Fourier transforms of the quantities defined above with the same symbols, just changing the domain of definition from time $t$ to frequency $\omega$. Since the Fourier transform of a convolution (filtering) converts to the

$$v_i(\omega) = h_i(\omega) \cdot [\delta(\omega) + \beta_i(\omega)]$$

which gives the Fourier transform (OFR), $\nu(\omega)$, of the EEG trial, $i$ at the frequency $\omega_i$ for any of the discrete frequencies numbered $f=1, \cdots, F$ measured. This expression has the following consequences:

- At any given frequency $\omega_i$, the OFR $\nu(\omega_i)$ is the simple addition of the evoked $h_i(\omega) \cdot \delta(\omega)$ and background activity $h(\omega) \cdot \beta(\omega)$, without any nonlinear interaction.
- There is no interaction between the OFR $\nu(\omega)$ for different frequencies. They are solely a function of system input at the same frequency.
- The OFR $\nu(\omega)$is a fixed multiple $h_i(\omega)$of system input $[\delta(\omega) + \beta(\omega)]$.

In the second half of the 20th century, there were already signs that this cozy linear system analysis was insufficient. In a seminal book, it was precisely Wiener (1958) who stressed that linear behavior was only a first approximation to describe real systems whose behavior was most probably nonlinear. Wiener generalized the approach of Volterra to describe this type of system. This “Weiner-Volterra expansion” can be informally described as a polynomial of time-dependent functions in which the linear term is only the first term. An excellent description of this theory can be found in (Billings, 2013). It is this expansion that is the theoretical underpinning of the nonlinear properties of complex systems.

Başar was aligned with Wiener’s viewpoints, and he repeatedly highlighted that linear methods (such as Fourier analysis) were only the first approximation to nonlinear brain dynamics. Usable nonlinear time series analysis methods only started to be available in the early ‘90s and were enthusiastically adopted at that time by Başar. However, till then, he was forced to use linear methods in frequency or time-frequency domain. Instead of being forced to “think linearly” it is precisely one of his main achievements to identify nonlinear brain dynamics by the properties of the EEG in the frequency domain. With the benefit of hindsight, we can now understand more formally the relation of frequency domain properties and the full Wiener-Volterra structure of nonlinear dynamics as we will now explain.

The OFR of a nonlinear system (Lang and Billings, 1997), which is based on the Wiener-Volterra expansion, details the effects of the nonlinearities of a system on the Fourier transform of its output. In other words, the effect of the nonlinearities on the linear analysis of systems. Eq. (1.3) generalizes to:

$$\nu(\omega) = \sum_{k=2}^{\infty} \sum_{l=1}^{\infty} \sum_{l=1}^{\infty} h_i(\omega) \cdot \beta(\omega) \prod_{l=1}^{k} \left[ h(\omega) \cdot \beta(\omega) \right] \tag{1.3}$$

Imposing as this formula is, one only need to notice the following

- For the OFR of linear systems the contribution of all the higher-order terms zero, in which case the OFR formula reduces to Eq. (1.3).
- The higher-order terms of the OFR can involve products of both the stimulus and the background activity at certain frequencies. This may produce an interaction between the two.
It is also possible for certain systems, that these products may induce additional activity in the OFR at frequencies that are multiples of those in signal or background (harmonics).

For severely nonlinear systems, OFR activity may appear at frequencies that are fractions of the input of frequencies (subharmonics).

We shall now see how Erol Başar—using at first only linear signal analysis methods—identified, and then theorized on this type of nonlinear phenomena. In passing, we will mention interactions with his friends and colleagues.

3.3. Interaction between event-related activity and ongoing EEG

Erol Başar was the first to point out the relationship between the frequency of EEG and Evoked Potential (EP) (Başar et al., 1976). By postulating the brain as a set of coupled oscillators it was possible to predict the occurrence of EPs. Başar stated that the self-oscillating system (brain) should be considered as an important feature of nonlinear dynamics and that this nonlinear pattern could provide insight into the doubling or tripling effect of EPs as the reaction of a self-organized dynamical system (Erol Başar, 1972b). Başar’s understanding of the use of the physical theories to explain the experiments leads to an empirically based theory of oscillatory neural assemblies, which considers the “real” activity of the brain to be its oscillations. In this framework, brain oscillations are not an epiphenomenon but reflect the processing of information in the brain (Karakas et al., 2000a, 2000b).

This study investigated the contribution of the delta and theta responses to two components of the event-related potential (ERP) waveform, the N200, and P300, employing multiple experimental paradigms. The systematically varying task conditions helped bring out the relations between the neuroelectric responses ERPs and event-related oscillations (EROs) and cognition. The data showed that the morphology of the ERP components for different experimental paradigms represented a specific pattern of superposition of the delta and theta oscillatory responses. As the main conclusion, they discussed the cognitive correlates of the oscillatory responses. The results were evaluated based on the superposition principle and the theory of oscillatory neural assemblies. More specifically, they concluded that “the delta and theta oscillations underlie both the N200 and P300 components. ERP components are the end products of a specific superposition of oscillations in various frequency bands. Thus, the basic phenomenon of brain neuro-electricity is not the ERP but the brain’s oscillatory responses, EROs”. In that sense, EROs are not an epiphenomenon (It is not a secondary phenomenon that occurs alongside or in parallel to a primary one).

This was followed by work that reinforced the idea of the interplay between non-phase locked oscillations (The genesis of human event-related responses explained through the theory of oscillatory neural assemblies (Karakas et al., 2000a, 2000b). The main objective of this paper was to study the role of delta and theta responses to N200 and P300 ERP components that are recorded from two topographical sites (Fz and Pz) under the known experimental paradigms (mismatch negativity and oddball) that trigger different cognitive processes for the respective task performances. Results showed that the theta response is related to a complex set of cognitive processes whereby selective attention becomes focused on a task-relevant template that is maintained in short-term memory. However, it was found that the theta response can be also obtained in response to inadequate stimulus and upon bimodal sensory stimulation (Basar, 1998; Başar-Eroglu et al., 1992; Klimesch, 1999) On the other hand, the study found that the delta

Fig. 8. EEG responses to Steady State stimulation at different frequencies. The fundamental frequency (stimulus freq. = response freq.) be as well as a first and second harmonic. Alpha (around 10 Hz) responds to many stimulation frequencies and preferably at intersections with the subharmonics. An enhanced response to 39 Hz stimulation can be observed. The figure was taken from Herrmann (2001b).

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J. Gulín-González et al.
response contributes to the amplitude at the P300 latency and that it varies in amplitude according to relevant task-respecting that demands conscious stimulus evaluation and memory updating. Delta response thus represents cognitive efforts that involve stimulus-matching and decision for the response to be made. The main outcome of this paper was that the oscillatory activity of the brain is a valid index of the brain’s cognitive processing.

The demise of the traditional deterministic view has been underscored by the work of Makeig who coined the term “event-related brain dynamics” (Makeig et al., 2004) ERD.

Based on Başar’s pioneering work, David et al. (2006) demonstrated theoretically the relationship between the mechanisms generating ERD. In a study (David et al., 2006), the neuronal transients were generated by changing neuronal input (a dynamic mechanism) or by perturbing the systems coupling parameters (a structural mechanism) to produce induced responses. David’s study, the influence of high-order kernels or equivalently generalized transfer functions means that a given frequency in the input can induce a different frequency in the output. A simple example of this would be squaring a sine wave input to produce an output of twice the frequency (see Fig. 9).

A revolutionary idea of Başar was the development of the Brain-Body-Mind Oscillations concept. This idea was discussed in the research report “Oscillations in ‘brain-body-mind’—A holistic view including the autonomous system” (Başar, 2008). Brain and body oscillation are synchronized in different frequency bands. Brain and the autonomous systems have three classes of oscillations: the higher frequency band (> 40 Hz), slow oscillations in the range of EEG frequencies, and ultra-slow oscillations in the frequency between 0.001 Hz and 1 Hz (called the rhythms of peristaltic organs and vasculature). Experimental results (Barman and Gebber, 1993) suggest the existence of oscillatory links in the brain, spinal cord, and in all organs of the body. However, as is mentioned by Başar is exceedingly difficult to establish a concrete model for the one-to-one connections and synchronization of oscillations between all parts of the body. Thus, he proposes a Gedanken-model based on the concept of globally coupled oscillators to elucidate the oscillatory dynamics of the brain/body. In (Başar, 2010a) he proposed two ideas: (1) that alpha oscillations are not “noise” signal, and (2) that the fundamental role of brain oscillations in brain-body function will be a difficult and protracted problem. Başar returned to these ideas in the book Brain-Body-Mind in the Nebulous Cartesian System: A Holistic Approach by Oscillations (Başar, 2010a) Recently, Klimesch (Klimesch, 2018) has also discussed the idea of the brain-body oscillations, demonstrating the legitimacy of Başar’s proposals.

During the last years, Başar focused the attention on the application of brain oscillations to the study of brain pathologies and cognitive disorders. He studied the changes in brain oscillations in patients with Alzheimer’s, schizophrenia, bipolar disorders, mild cognitive impairment, attention-deficit hyperactivity disorder, alcoholism, and different genetic disorder. A very complete review of this topic was published by Başar in 2008 (Başar and Güntekin, 2008).

3.4. The appearance of harmonics (and subharmonics)

Evidence for nonlinear behavior of brain responses started to appear in the 1960s. One of first was Van der Tweel (Van Der Tweel and Lutel, 1965) who used a light signal modulated sinusoidally, the steady-state evoked potential (SSVEP), to obtain the transfer function of the visual system. He demonstrated the appearance of harmonics, leading him to postulate a rectifier response in the visual pathway. However, before 1970, only a small number of authors had investigated the behavior of the EEG-response using sinusoidally modulated light or sound signals.

Erol Başar in a seminal paper, “A Study of the Time and Frequency Characteristics of the Potentials Evoked in the Acoustical Cortex” (Başar, 1972a). He said: “The difficulty results from the requirement for evoked responses to sinus signals of over at least three decades of stimulation frequencies, evoked responses in each stimulation frequency being averaged using at least 200 stimuli. Another difficulty comes from the fact that some brain activity stages have a duration of only a few minutes during which the application of sinusoidal stimuli of different frequencies is impossible.” In that period, researchers preferred to obtain evoked responses in the time domain. It was suggested that it was more useful and enough to analyze and characterize the transient response terms of peak (wave) latencies, peak shapes, and directions by using tone bursts, clicks, or light flashes as stimuli. Also, in this first contribution, Başar proposed to transform the evoked potentials in the time domain to the frequency domain. Thus, it was possible to study a wide frequency scales of the frequency characteristics and brought a new concept to the understanding of the time course of evoked potentials. Under the conditions of the performed experiment, he found that the nonlinearities, distortions of the response against the pure sine-wave stimulus, were very strong. His results showed that the responses evoked with sinusoidally modulated light signals are nonlinear. He proposed a semiempirical method that allowed the mathematical evaluation of the characteristic frequency from the transient evoked potentials and, consequently, shortens the experiments and allowed the widening of the frequency scale. Besides, it enabled him to describe the brain as a dynamical system, whose state is changing continuously. In words of Başar (Başar, 2010a) “brain dynamics has features of physical dynamics systems.”

A particularly clear and recent experimental demonstration of the non-linear behavior of the brain processes was given by the experiments by Herrmann (2001a). Oscillations in the EEG are classified according to their relation to stimulation and can be spontaneous, evoked or induced according to Başar’s classification (Başar-Eroglu et al., 1996). Herrmann experimented with human subjects, presenting a flickering light at frequencies from 1 to 100 Hz in 1-Hz steps. He found that the event-related potentials exhibited steady-state oscillations at all frequencies up to at least 90 Hz. The steady-state potentials exhibited clear resonance phenomena around 10, 20, 40, and 80 Hz. Harmonic and subharmonic were observed (Fig. 9). Response frequencies which are integer multiples of the stimulus frequency are also evoked by the flicker. Also, subharmonic oscillations were seen at the intersection with the 10-Hz alpha response. It is noteworthy that this result is precisely the confirmation of the non-linear behavior of brain processes already demonstrated by Başar in freely moving cats with implanted electrodes (Röschke and Başar, 1988).

3.5. Nonstationarity of event-related activity

Wavelet entropy was another important topic in which Başar used physical ideas to explain the behavior of the brain’s electrical signals. In 2001, he published the contribution “Wavelet entropy: a new tool for analysis of short duration brain electrical signals” (Rosso et al., 2001). In this paper, he described the use of quantitative parameters derived from the orthogonal discrete wavelet transform applied for the analysis of short duration brain electrical signals. The Wavelet Entropy (WE) in contrast with the spectral entropy has the following advantage (Rosso et al., 2001). (i) WE is capable of detecting changes in a nonstationary signal due to the characteristics localization of the wavelet transform. (ii) In contrast to dimensions and Lyapunov exponents or dimensionality and chaoticity measures, the computational time of WE are significantly shorter since the algorithm involves the use of fast wavelet transform in a multiresolution framework. (iii) Contaminating noises contributions can be easily eliminated; and (iv) The WE are parameter-free (It is very important). Also, this study demonstrated that WE are physiologically meaningful since it differentiated specific physiological brain states under spontaneous or stimulus-related conditions. Also, they found a decrease in the WE in the post-stimulus epoch, “indicating a more rhythmic and ordered behavior of the EEG signal compatible with a dynamic process of synchronization in the brain activity”. Time evolution parameters derived from WE were used to identify the time
localizations of dynamic processes in the post-stimulus epochs. The WE gave new information about EEG/ERP signals in comparison to the one obtained by frequency analysis or other standard methods. Another result of this paper was the use of the quantifiers based on time–frequency. It should be emphasized that this paper established a detailed neurophysiological links between all measured parameters and brain processes.

3.6. Nonlinear dynamics

Another important field that Başar thought very relevant to brain dynamics is was that of nonlinear dynamics, in particular, “chaos theory”. Chaos is understood as irregular fluctuations deterministic nonlinear equations. Nonlinear methods and chaos theory have been critical for a complete understanding of the EEG signals. The introduction of concepts as attractors or correlation dimension is commonly used by neuroscientists today. Since his first published study (Erol Başar, 1972a), the contribution of Başar in this branch of neurophysiology was decisive. Applications of these methods allowed to explain interesting phenomena and processes in the brain. Attractor as a concept to understand the EEG was studied in several papers by Başar and collaborators (Röschke and Başar, 1990; Röschke and Başar, 1985). Also, chaotic EEG behavior during slow-wave sleep in the cat cortex and hippocampus was studied by Başar (Röschke and Başar, 1989).

Closely related theoretical work started about the same time, with the introduction of neural mass models (Lopes da Silva et al., 1974; Wilson and Cowan, 1972) showed that the appearance of harmonics in the EEG alpha rhythm was dependent on the sigmoid response of these masses. Particularly it is good to mention the scientific exchanges between Başar and Lopes da Silva and especially with Walter Freeman (Başar, 2016; Freeman, 1998). These ideas were put into a statistical estimation framework by (Valdes et al., 1999). This type of work later evolved into the now well-known Dynamic Causal Modeling, a recent example being (Daunizeau et al., 2012).

3.7. Theoretical outlook

We finally must mention Erol Başar’s provocative thoughts about the interplay of Theoretical Physics and Neuroscience. Quantum mechanics is the physical theory of the probability and the uncertainty, contrary to the deterministic paradigm of the Newton mechanics. As we previously discussed, the impact of quantum theory on the brain processes is strongly discussed up to the present. In this context, the Başar’s position was favorable to the influence of this theory for neurosciences. Here, we only mention some examples of this support. Heisenberg’s uncertainty principle was used by him to explain the continuous changes of the brain activity due to the stimulation of the brain with a sequence of cognitive working memory inputs (Başar, 2010a). He frequently used the famous “model of thought”, that is the microscope theory (it referred to the impossibility to construct a microscope with a resolution which allows to directly observe the electron, due to that the interaction between the electromagnetic radiation and the electron modifies the state of the electron). This theory was experimentally demonstrated by Foot in 1994 (Foot, 1994) According to Başar’s interpretations of the uncertainty in brain research “when the brain is stimulated by a sequence of peripheral stimulations the spontaneous activity of the brain does incessantly change.”(Başar, 2010b). On the other hand, a system of neurons can behave in a probabilistic manner. Mostly, neurons may or may not fire. Thus, once a neuron fires, the experimenter is no longer able to excite the neuron at the beginning of the process. Experiments in humans have demonstrated this fact, as a confirmation of the probabilistic behavior of the brain structures (Başar, 2010a) Other interesting and polemic...
theoretical appropriation of Başar, related to the concepts of brain dynamics and quantum brain, was the use of the S-Matrix (Başar, 1983; Erol Başar, 1998) (derived from the S-theory of Heisenberg in quantum mechanics) to some aspects included in theoretical psychology (Hayek, 1952; FA, n.d.) which consider the whole history of an organism. Metaphors related to the S-matrix and Feymann’s diagrams were proposed by him to represent the complex brain’s processes (Başar, 2009). The generalization of these quantum analogs led to him to defend the concept of the quantum brain, this concept was of great for him during his last few decades (Başar, 2010b).

3.8. Erol Başar’s legacy

When Kuhn published in 1962, the iconic and controversial book The Structure of Scientific Revolutions (Kuhn, 1970), he introduced into science and philosophy the concept of “paradigm”. According to Kuhn, the evolution of science passes through different stages (Fig. 1). “Normal” science is determined by adherence to what Kuhn called a “paradigm”. This paradigm is a set of theories that provide standard tools to the scientist to solve the problems, focusing on research efforts. As explained in the introduction, the appearance of “anomalies” is followed by a scientific revolution.

Until the 60’s decade of the past century, normal science in neurophysiology included a purely phenomenological explanation of the brain processes and, particularly, in electrophysiology the absolute separation between “evoked” and “background activity”. The underlying conceptual model was classical linear systems theory that explained evoked potentials as the response through a linear transfer function to a deterministic transient superimposed on independent background noise. It must be understood that, at this time, the application of physical theories and mathematics in observed neurophysiological findings were very limited. To talk about a paradigm in neurophysiology is a risky issue, but it is possible to allow this term to the introduction of the nonlinear-analysis, physical theories, and mathematics to the explanation of the brain functioning. Brain dynamics changed the image of the brain as a “static” organ and has allowed the understanding of its complex nature (Fig. 1). A significant number of scientists: N. Wiener, W. J. Freeman (1927–2016), E. Roy John (1924–2009), Fernando Lopes da Silva (1935–2019), and other contributed to the application of nonlinear analysis to neurophysiology. They were architects of the change of world view in the interaction of Event-Related and Background activity. As we have pointed out new branches of the brain studies appeared (e.g. quantitative electroencephalography and neurometrics). The entire labor of Erol Başar made him one of the main contributors to a major paradigm shift: from linear to non-linear systems analysis of electrophysiological phenomena.

A crucial point in this transition was the meeting in Havana (John, 1989a, 1989b) in which Başar, as well as many others mentioned in this review (Roy John, Freeman, Lopes da Silva, Van der Tweel, Haken and many others), had the opportunity to discuss all the issues mentioned, and to chart the new scientific revolution.

At the end of his life Erol, his deeper knowledge of the physiology of the brain led him to focus attention on the integration of all his knowledge about the brain and the understanding of the mind functioning. He assumed the mind as an integration of several physiological and psychological entities (Başar, 2010a).

4. Conclusions

The analysis of Erol Başar’s scientific production covered more than 40 years, a critical period for the development of theory in the neurosciences. His intense scientific activity is evidenced by 278 contributions in peer review journals, several books, and more than 7500 citations. At the same time, he created a strong network of collaboration among scientific groups of different countries. Başar’s main research topics were focused on Brain Dynamics, which is a mechanic underlying the brain, based on inspiration from physics, mainly Newton Dynamics, quantum mechanics, and nonlinear dynamics. Application of physical and nonlinear random system theories provided Başar with the tools for a deeper understanding of brain functions as a complex system. The first and important contributions of Başar were transforming the analysis of the evoked potentials in the time domain to the frequency and later time/frequency domain. Therefore, it was possible to study a wide scale of the EEG frequency characteristics and brought a new concept to the understanding of the temporal course of evoked potentials. He explored the application of physics theories to explain the physiology of the brain in several papers and books. Particularly important for his work was the application of the quantum theory to explain the brain dynamics, through the idea of the quantum brain. In our opinion, it is possible to consider to Erol Başar as one of the founders in the introduction of the non-analysis, physical theories for explaining brain function. All these contributions made of Başar a protagonist of the revolution in neurophysiology. We would like to conclude this article with the words of Walter J. Freeman, evoked by Başar in the Obituary that he wrote in memoriam of his friend in this journal: “Soon, one-third of neuroscience publications will be related to brain oscillations” (Başar, 2016).

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